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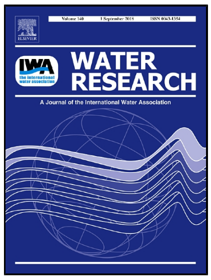

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Accounting for variation in rainfall intensity and surface slope in wash-off model calibration and prediction within the Bayesian framework

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Abstract

Exponential wash-off models are the most widely used method to predict sediment wash-off from urban surfaces. In spite of many studies, there is still a lack of knowledge on the effect of external drivers such as rainfall intensity and surface slope on the wash-off prediction. In this study, a more physically realistic "structure" is added to the original exponential wash-off model (OEM) by replacing the invariant parameters with functions of rainfall intensity and catchment surface slope, so that the model can better represent catchment and rainfall conditions without the need of lookup table and interpolation/extrapolation. In the proposed new exponential model (NEM), two such functions are introduced. One function describes the maximum fraction of the initial load that can be washed off by a rainfall event for a given slope and the other function describes the wash-off rate during a rainfall event for a given slope. The parameters of these functions are estimated using data collected from a series of laboratory experiments carried out using an artificial rainfall generator, a 1 m^2 bituminous road surface and a continuous wash-off measuring system. These experimental data contain high temporal resolution measurements of wash-off fractions for combinations of five rainfall intensities ranging from 33-155 mm/hr and three catchment slopes ranging from 2-8 %. Bayesian inference, which allows the incorporation of prior knowledge, is implemented to estimate parameter values. Explicitly accounting for model bias and measurement errors, a likelihood function representative of the wash-off process is formulated, and the uncertainty in the prediction of the NEM is quantified. The results of this study show: 1) even when OEM is calibrated for every experimental condition, NEM's performance, with parameter values defined by functions, is comparable to OEM. 2) Verification indices for estimates of uncertainty associated with NEM suggest that the error model used in this study is able to capture the uncertainty well.

Keywords: Sediment wash-off, Model structure, Bayesian framework, Autoregressive error model

1. Introduction

Urban surface sediment's ability to act as a transport medium to many contaminants makes it one of the major source of pollutants in an urban environment (Collins and Ridgeway, 1980; Guy, 1970; Lawler et al., 2006; Mitchell et al., 2001). Hence there is an increasing interest in being able to better predict the sediment wash-off from urban surfaces. But, modelling sediment wash-off is not a straightforward exercise as it requires the understanding of complex interactions between external drivers with a highly variable nature such as rainfall, catchment surfaces and particle characteristics (Deletic et al., 1997; Egodawatta and Goonetilleke, 2008; Sartor and Boyd, 1972). Currently, the most widely used wash-off models are originally developed using laboratory experiments and consequently include empirical parameters without clear physical interpretations. The exponential wash-off equation (Eq.1) proposed by

Sartor and Boyd (1972) is one such model whose performance is highly dependent on the accurate estimation of parameter k:

$$w_t = w_0\left(1 - e^{-kR_t}\right) \qquad (1)$$

Where w_t is the total transported sediment load up to time *t*; w_0 is initial load of sediment on the catchment surface; R_t is cumulative rainfall depth at time *t,* i.e. $i_t t$ where i_t is average rainfall intensity over time *t,* ; and *k* is an empirical wash-off coefficient.

Equation 1 has widely been used in several software packages (e.g. SWMM) with or without modifications (e.g. Zug et al. 1999; Huber and Dickinson 1992). Since, rainfall is the main driver the wash-off process (Deletic et al., 1997; Egodawatta et al., 2007; Sartor and Boyd, 1972; Shaw et al., 2010), understandably most of these modifications are focused on the effect of rainfall. Recently, Egodawatta et al. (2007) suggested an introduction of a 'capacity factor' which gives a more physical interpretation to the empirically calibrated original model shown in Eq.1. According to Eq.1, if the rainfall continues for long enough regardless of the rainfall intensity, it can wash off all the sediment available at the beginning of the event. In other words, the maximum wash-off fraction ($^{w_t}/_{w_0}$) is always one. But Egodawatta et al. (2007) showed that a storm event has the capacity to wash-off only a fraction of sediments available and once this maximum fraction is reached the wash-off becomes almost zero, even though a significant fraction of sediment is still available on the surface. They suggested the introduction of an additional term referred to as the 'capacity factor' (C_F) to replicate this finding in the model equation as shown is Eq. 2

$$\frac{w_t}{w_0} = C_F\left(1 - e^{-kR_t}\right) \qquad (2)$$

Although the above modification was shown to be a meaningful refinement, C_F was investigated against rainfall intensity in isolation in Egodawatta et al. (2007). Muthusamy et al. (2018) further showed that C_F also varies with catchment surface slope in addition to rainfall intensity. Despite surface slope's direct impact on mainthe underlying process of sediment wash-off which are impact energy from rainfall drops (Coleman, 1993) and shear stress from overthe land flow (Akan, 1987; Deletic et al., 1997), there is a clear lack of attention given to surface slope in thge literuture. Results from Muthusamy et al., (2018) showed that the surface slope has a siginificanlt effect on the wash-off load and this effect should not be neglected in the prediction of wash-off.

In spite of the modifications suggested by various studies including Egodawatta et al. (2007) and Muthusamy et al. (2018), the calibration parameters k and the newly introduced C_F still need to be calibrated for the conditions of each catchment. In general, this is achieved by using a combination of look up tables/charts and interpolation/extrapolation of existing data. However, with the absence of such commonly accepted look up tables/charts, the modellers are forced to use a constant values for parameters regardless of catchment conditions. This calls for an alternative and a more transparent way of estimating the calibration parameters.

Furthermore, none of the abovementioned studies includes any information on the uncertainty in the estimation of the calibration parameters and their dependency structure which needs to be accounted in the prediction of wash-off using these parameters. Although adequate treatment of propagation of uncertainties in model prediction is a currently heavily researched area in hydrology, there are only a few studies on uncertainty related to wash-off modelling (e.g. Sage et al. 2016; Dotto et al. 2012). In this regard, Dotto et al. (2012) compared a number of uncertainty techniques applied in urban water stormwater quality modelling and found that a Bayesian approach, although computationally demanding, to be one of the preferable uncertainty assessment technique. A Bayesian approach helps to identify different sources of uncertainty such as parameter uncertainty, model bias and measurement noise and consequently, helps to separately analyse them, though this requires knowledge about the error process (Dotto et al., 2012). In this regard, Sage et al. (2016) discussed the consequences of using a wrong error model in the prediction of uncertainty in wash-off modelling and called for more attention to be paid for the selection of error model.

Considering the above research gaps in the current modelling approach of sediment wash-off, this study aims:

a) To add a more physically realistic "structure" to Eq. 2 by replacing the calibration parameters with functions of external drivers associated with catchment surface and rainfall characteristics and compare its performance with the original model.

b) To identify different sources of uncertainty associated with the new wash-off model developed in (a) and estimate reliable prediction intervals using a suitable error model

2. Material and Methods

2.1 Wash-off Data

Data used in this study were collected from a series of laboratory experiments carried out using an artificial rainfall generator, a 1 m^2 bituminous road surface and a continuous wash-off measuring system (Fig.1). This data contain sediment wash-off data measured against different combinations of rainfall intensity, catchment surface slope and initial sediment load. The road surface was prepared using bituminous asphalt concrete and had a mean texture depth index of 0.4 mm. D_{10}, D_{50} and D_{90} of the sand used in the experiment are 300 μm, 450 μm and 600 μm respectively. Five intensities ranging from 33-155 mm/hr, four slopes ranging from 2-16 % and three initial loads ranging from 50 - 200 g/m^2 were tested in these experiments. For more details on the experimental setup, selection of experimental conditions and data collection the readers are referred to Muthusamy et al. (2018). As reported in Muthusamy et al. (2018) the effect of initial load on wash-off process was found to be negligible. Hence in this study, experimental results from a constant initial load of 200 g/m^2 as presented in Fig. 2 were used. This figure shows the variation of cumulative wash-off fraction ($F_w = {}^{w_t}/_{w_0}$) against rainfall intensity and surface slope.

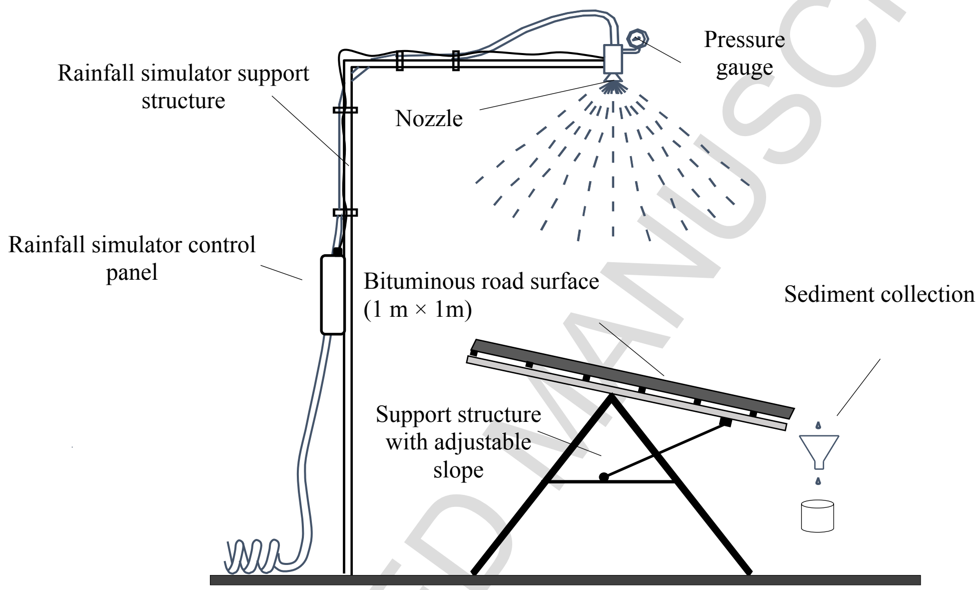

Figure 1. Sketch of the experimental setup

Note that the 16% slope was eliminated from the data, given that such slopes on road surfaces are extreme scenarios and exist only in rare locations. For example, the Department of Transport in the UK suggests a maximum gradient of 10% for roads other than in exceptional circumstances (Manual for Streets, 2009). Since one of the aims of the study is to develop a single model with a fixed set of parameters, the inclusion of results from such an extreme scenario in the calibration may affect the performance of the model for more general cases.

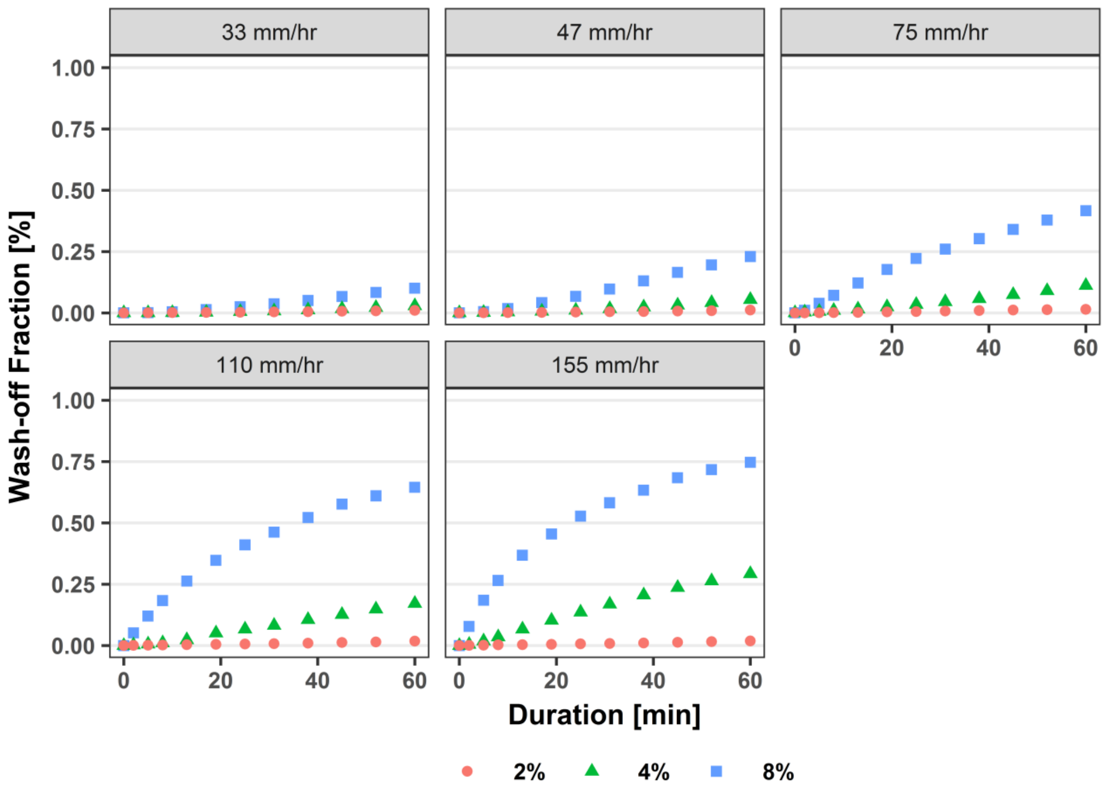

Figure 2: Selected results from Muthusamy et al. (2018): Variation of wash-off fraction for different combinations of rainfall intensity and surface slope

2.2 The modified wash-off model structure and its rationale

The main objective is to replace the calibration parameters in Eq. 2 with functions of surface slope and rainfall intensity, consequently adding a more physically realistic structure to the model. This should make the model robust to new combinations of rainfall intensity and surface slope. To do so, the properties of the model that are sensitive to such parameters need to be identified and understood. From Eq. 2 there are two parameters which define the characteristics of a wash-off curve. The first parameter, C_F, defines the highest wash-off fraction for a given combination of rainfall intensity and a slope. The second, *k,* defines how fast the wash-off curve reaches the maximum fraction for a given surface slope and rainfall intensity, and hence reflects the erosion rate from the catchment surface. Hence, C_F and k were proposed to be replaced with functions of surface slope and rainfall intensity, as shown in Eq. 3 and Eq. 4.

$$C_F = c_1 {i_m}^{c_2} s^{c_3} \qquad (3)$$

$$k = c_4 s \qquad (4)$$

Where c_1,.., c_4 are constants[1], i_m is the representative rainfall intensity of a rainfall event (e.g. in this case the constant rainfall intensity set during the experiment, please refer to section 3.4 for discussion on the use of representative rainfall intensities), s is the catchment surface slope. The following criteria were considered when defining Eq. 3 and Eq. 4, while also trying to keep these functions as simple as possible to reduce the number of constants:

- C_F– as explained before C_F is a capacity factor which defines the maximum fraction from the initially available sediment that can ever be washed off from a rainfall event for a given slope. Hence, C_F ranges from 0 to 1 and increases with both surface slope and (representative) rainfall intensity of the event. When either of the representative rainfall intensity or slope is zero C_F is zero.
- $k - k$ defines the wash-off rate and it also increases with rainfall intensity and surface slope. But it should be noted that R_t in the exponential term is cumulative rainfall depth at time *t*, i.e. $i_t t$ which is already a function of average rainfall intensity over time *t*, i_t. Hence k was taken as a (linear) function of slope only. The complete exponential term reads as $c_4 s i_t t$ which is function of both rainfall intensity and surface slope.

[1] Although c_1,.., c_4 are constant, in Bayesian inference they are referred to as model parameters to aid the readers follow the procedure easily.

Hereafter this new exponential model will be referred as NEM and the original exponential model as shown in Eq. 1 will be referred as OEM.

2.3 Estimation of model parameters and associated uncertainty

Bayesian inference was used to estimate the parameter probability distribution, which allows prior knowledge on the parameters to be incorporated in the estimation and also formally quantifies uncertainty in the estimation (Dotto et al., 2012; Freni and Mannina, 2010; Del Giudice et al., 2013). In addition, it also helps to capture the dependence structure between parameters (Dotto et al., 2012). Bayesian inference requires the definition of the likelihood function and the prior distribution of the parameters.

2.3.1 The likelihood function

In addition to finding the best estimate of the parameters, we are also interested in the uncertainty associated with the parameter estimation and consequently the uncertainty in the prediction of the wash-off fraction. One way of doing this is to include the error terms which represent the dominant sources of uncertainty explicitly in the likelihood function. We used an error model which accounts for errors due to the model structural deficit (model bias, $\boldsymbol{B_M}$) and measurement noise ($\boldsymbol{E}$). $\boldsymbol{B_M}$ is modelled as an autoregressive stationary random process and $\boldsymbol{E}$ modelled as an independent identically distributed (IID) normal noise. Hence, an observed output, $\boldsymbol{Y}_o$ can be formulated as

$$\boldsymbol{Y}_o(\boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\psi}) = \boldsymbol{y}_m(\boldsymbol{x},\boldsymbol{\theta}) + \boldsymbol{B}_M(\boldsymbol{x},\boldsymbol{\psi}) + \boldsymbol{E}(\boldsymbol{\psi}) \qquad (5)$$

Where x is the external drivers, $\boldsymbol{\theta}$ is deterministic model parameters, $\boldsymbol{\psi}$ error model parameters and $\boldsymbol{y}_m(\boldsymbol{x},\boldsymbol{\theta})$ is deterministic model output. In this case, $\boldsymbol{Y}_o$ is observed wash-off fractions (F_w) and $\boldsymbol{y}_m$ is the deterministic model output predicted from NEM (f_w). $\boldsymbol{x}$ represents rainfall intensity and surface slope. $\boldsymbol{\theta}$ represents parameters $c_1,.., c_4$. $\boldsymbol{\psi}$ represents error model parameters $sd.B$, $sd.E$ and l in which $sd.B$ and l are standard deviation and the correlation length respectively that characterise the autoregressive stationary random process and, $sd.E$ is the standard deviation of the measurement noise. Given the error description of Eq. 5, we define $\boldsymbol{B}_M(\boldsymbol{x},\boldsymbol{\psi})$ as a multivariate Gaussian distribution with covariance matrix $\Sigma(\boldsymbol{x},\boldsymbol{\psi})$ and $\boldsymbol{E}(\boldsymbol{\psi})$ as independent, identical normal noise. Therefore, the analytic formulation of the likelihood function with *n* number of observation can be formulated as

$$P(\boldsymbol{y}_o \mid \boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\psi}) = \frac{(2\pi)^{-\frac{n}{2}}}{\sqrt{\det\left(\sum(\boldsymbol{x},\boldsymbol{\psi})\right)}} \exp\left(-\frac{1}{2}\left[\boldsymbol{y}_o - \boldsymbol{y}_m(\boldsymbol{x},\boldsymbol{\theta})\right]^T \sum(\boldsymbol{\psi},\boldsymbol{x})^{-1}\left[\boldsymbol{y}_o - \boldsymbol{y}_m(\boldsymbol{x},\boldsymbol{\theta})\right]\right) \tag{6}$$

The covariance matrix $\Sigma(\boldsymbol{x},\boldsymbol{\psi})$ was formulated using covariance in time calculated using OU process (Uhlenbeck and Ornstein, 1930). For hydrological applications, OU process assumed to be a simple description of underlying mechanisms leading to a decay of correlation in time (Del Giudice et al., 2013; Sikorska et al., 2012; Yang et al., 2007). A detailed description of the formulation of covariance matric using OU process can be found in Del Giudice et al. (2013). An autoregressive error model represents model structural deficit better than IID as it accounts for the "memory" in the error time series (Del Giudice et al., 2013). This autoregressive bias error model was originally suggested in other generic statistical applications (Bayarri et al., 2007; Craig et al., 2001; Higdon et al., 2004; Kennedy and O'Hagan, 2001) and later adapted for environmental engineering applications (Reichert and Schuwirth, 2012).

2.3.2 Prior distribution of parameters and constraints

Since the introduced parameters c_1,.., c_4 are all new, there is no previous estimation of the exact parameters, but a range for each parameters can be derived using our knowledge of the wash-off process, observational data, and the prior belief about values of C_F and k.

Values of c_4 were derived from previous estimations of *k* as c_4 equals to *k/s.* The list of *k* values derived from previous studies is given in Table. 1. From the table, the range of 0 – 10 were selected for k. In the absence of any information on slope in most of these studies same range for c_4 was used considering a minimum slope of 1%. Hence a uniform prior with the range 0-1000 was used as a prior distribution for c_4. A uniform prior distribution of model parameters can be used when there is not enough evidence available to choose a different type of distribution (Dotto et al., 2012; Freni and Mannina, 2010)

Table 1: *k* values from the literature

Reference	Land use/catchment type	Value k (mm^{-1})
Alley (1981)	Urban catchment	0.036-0.43
Nakamura (1984)	Various	0.05-10
Huber and Dickinson (1992)	General	0.04-0.4
Millar (1999)	Residential	0.21
Egodawatta et al. (2007)	Concrete and asphalt roads	5.6×10^{-4}– 8.0×10^{-4}

As discussed previously, the range of C_F is 0-1 because wash off fraction cannot be more than 1. This leads to the constraint $0 \le c_1 {i_m}^{c_2} s^{c_3} \le 1$ .. The implication of this constraint in the definition of prior probability is not straightforward as it involves three parameters, hence this constraint was used during the estimation of likelihood probability.

It is challenging to define prior distributions for the error model parameters ($sd.B, sd.E$ and l) especially in the case of wash-off modelling as examples from such applications in literature are currently lacking. Out of the three parameters, some information on the measurement noise represented by $sd.E$ can be obtained by frequentist tests, i.e. repeating the experiments sufficiently large number of times. But it is not always possible given the limitation in allocated resources and time. In the absence of much information on any of the error parameters, a uniform prior with the range from 0 to 1(= maximum wash-off fraction) was used for both $sd.B, sd.E$ and a uniform prior with the range of 0 – 200 min was used for correlation length. This range is selected as error correlation is expected to be insignificant beyond such time length.

2.3.3 Bayesian inference

Once the prior distributions (the probability of deterministic and error model parameter, $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ without considering the observed output, $\boldsymbol{y}_o$), $P(\boldsymbol{\theta},\boldsymbol{\psi})$, and the likelihood function (the probability of seeing the observed output, $\boldsymbol{y}_o$, as generated by a model with deterministic and error model parameter, $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$), $P(\boldsymbol{y}_o | \boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\psi})$, are defined, the posterior distribution of the deterministic and error model parameters (the conditional probability of $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$ once the observed output, $\boldsymbol{y}_o$ has been taken into account) can be formulated as,

$$P(\boldsymbol{\theta},\boldsymbol{\psi} | \boldsymbol{y}_o,\boldsymbol{x}) = \frac{P(\boldsymbol{y}_o | \boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\psi})P(\boldsymbol{\theta},\boldsymbol{\psi})}{\int P(\boldsymbol{y}_o | \boldsymbol{x},\boldsymbol{\theta},\boldsymbol{\psi})P(\boldsymbol{\theta},\boldsymbol{\psi})d\boldsymbol{\theta}d\boldsymbol{\psi}} \tag{7}$$

Since the direct analytical calculation of $P(\boldsymbol{\theta},\boldsymbol{\psi} | \boldsymbol{y}_o,\boldsymbol{x})$ is generally not possible, numerical techniques such as Markov Chain Monte Carlo (MCMC) simulations have to be applied to generate samples for this distribution. MCMC techniques generate a random walk through the parameter space which will converge to the posterior distribution. In this study, we used robust adaptive Metropolis MCMC sampler presented in Vihola (2012) which is implemented in an R package, *adaptMCMC* (Scheidegger, 2017).

2.4 Performance assessment

Experimental data with 2% and 8% slopes (two-thirds of the total data) were used for calibration of NEM and the data from the 4% slope (one-third of the total data) were used for verification. The optimal value of each parameter $c_1 \ldots c_4$ obtained during the calibration stage was then used for validation. Furthermore, the performance of NEM was compared against OEM during both calibration and validation stages. In the case of NEM, the k value was calibrated for each and every combination of surface slope and rainfall intensity during the calibration stage. Linear interpolation of these calibrated k values was then used to obtain new k values during the validation stage for a new surface slope condition.

In addition to deterministic prediction, prediction uncertainty of NEM was also obtained during both calibration and validation stages. Parameter and total predictive uncertainty (parameter uncertainty + model bias + measurement noise) were predicted by sampling from posterior multivariate distributions of parameters $c_1, .. c_4$. Parameter uncertainty was estimated by using deterministic model ($\boldsymbol{y}_m$ (x,θ)) runs and predictive uncertainty was estimated by using the deterministic model together with error model components.

3. Results and discussion

3.1 Model performance

Figure 3 shows the model output with the optimal values for c_1,.., c_4 (Table. 1) with maximum posterior probability density, i.e. the most probable values given the prior and observed data. . It can be seen from Fig. 3 that with calibration data, NEM with fixed values of parameters c_1,.., c_4 , corresponding to the maximum posterior probability density, performs as well as the OEM which was calibrated for each and every combination of surface slope and rainfall intensity separately. From Table 2, it can be seen that the difference in sum of root mean square error ($RMSE_{OEM}$ - $RMSE_{NEM}$) from the ten calibrated set of data is -0.07 (Wash-off fraction). However, the robustness of NEM over OEM can be seen during the verification stage where the NEM performs better than the OEM in several cases. The difference in sum of root mean square error ($RMSE_{OEM}$ - $RMSE_{NEM}$) from 5 sets of data during verification stage is 0.09 (Wash-off fraction). The drawback with OEM is that for a set of new catchment conditions where OEM has not been calibrated before k value needs to be calculated using interpolation/extrapolation. This might lead to the underperformance of OEM during validation stage as shown in the Fig. 3. Considering the overall performance, the NEM with only 4 parameters (c_1,.., c_4) performs better than OEM with 15 parameters ($k_1,\ldots,k_{15}$). Hence, the NEM does not only avoid the need of interpolation to predict the calibration parameter values, it also performs as well as the calibrated OEM.

Table 1: Optimal values of constants of Eq. 3 and Eq. 4

c_1	c_2	c_3	c_4
3.99	0.672	1.99	0.208

Table 2: Performance of OEM and NEM

Model	Sum of root mean square error (RMSE)	
	Calibration	Verification
OEM	0.11	0.20
NEM	0.18	0.11

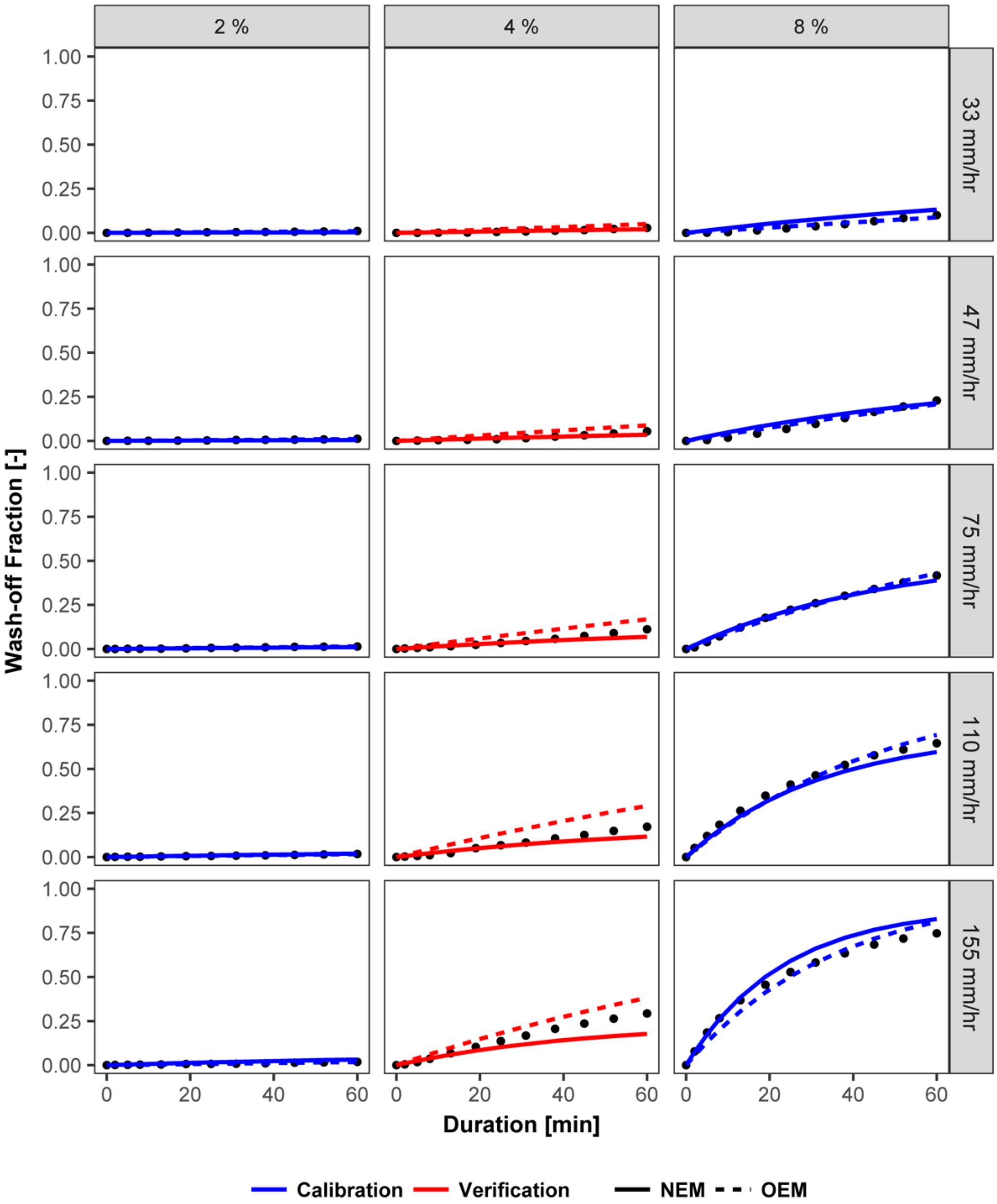

Figure 3: Comparison of the model performance

3.2 Parameter distribution and correlation

This section discusses the posterior distribution of parameters and their multivariate behaviour. Figure 4 shows posterior distributions and a bivariate matrix of the deterministic and error model parameters. The most likely value of $sd.B$ and $sd.E$ are 0.02 (2%) and 0.002 (0.2%) respectively, showing that most of the uncertainty in the wash-off estimation can be explained by the model bias and that uncertainty due to measurement noise is negligible. Although these are approximate representations of the actual system and corresponding uncertainty, we believe that the experiments were conducted with as high a quality as possible. This is one of the reason why a road surface as small as 1 sq.m was selected as it gives a better control over the experimental set-up. For example the smaller surface area keeps the spatial variability of the rainfall to the minimum. Furthermore, it also keeps the sediment loss during the experiment to insignificant. The maximum sediment loss observed during an experiment was less than 2% which is an indication of the good quality control.

Looking at the bivariate plots, there is a strong positive correlation between parameters $c_1$and c_3 which indicates that these two parameters compensate each other in order to maximise the posterior probability. This can also be seen between parameters c_2 and c_3, but to a lesser extent. Similarly, the strong positive correlation between $sd.B$ and l means that these parameters compensate each other in order to fit the autoregressive error model B_M. Bayesian inference helps resolve such identifiability issues by allowing for informative priors. Therefore, for real cases, where we have reasons to believe that one of the two parameters should be more constrained, the other parameter value will automatically come out to be constrained after joint inference.

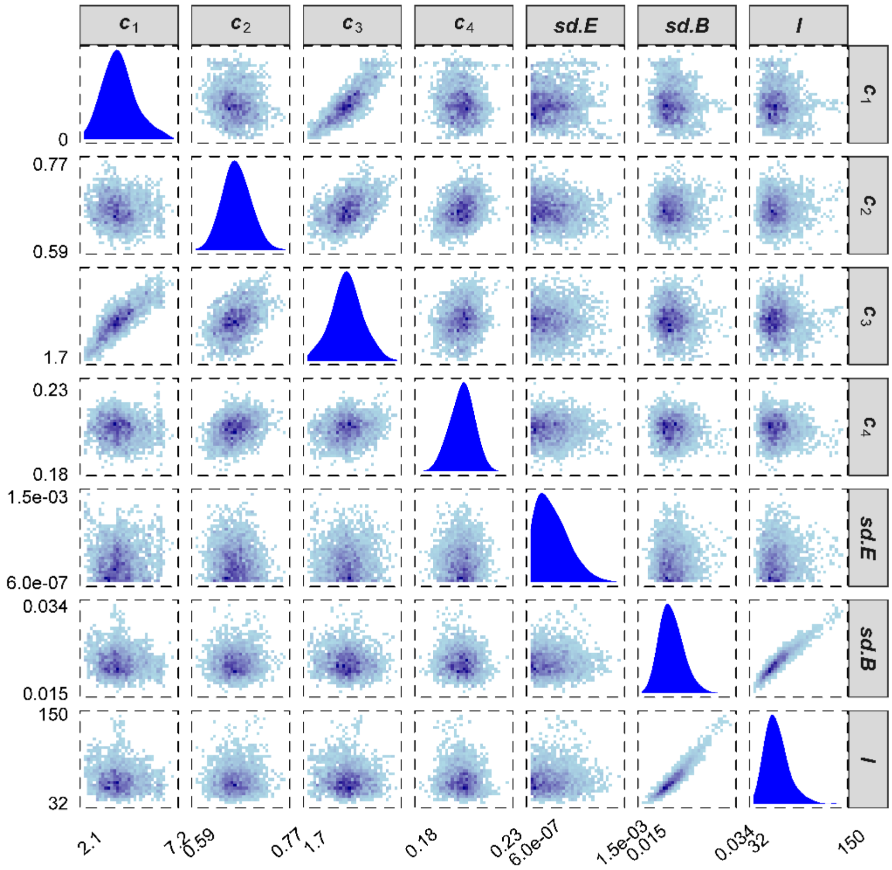

Figure 4: Parameter distribution and bivariate correlation

3.3 Estimation of parameter and predictive uncertainty

Figure 5 shows the uncertainty associated with the estimation of the wash-off fraction. Parameter uncertainty was estimated by using deterministic model ($\boldsymbol{y}_m$ (x,θ)) runs and predictive uncertainty was estimated by using the deterministic model together with error model components. Since the latter also includes the uncertainty due to model bias and measurement noise these bands are wider than the parameter uncertainty. The total predictive uncertainty which accounts for parameter uncertainty, model bias and measurement noise accounts for ~ 0.1 (10%) uncertainty in the wash-off fraction. This constant trend of predictive uncertainty is a reflection of the fact that the error model used here is not explicitly input-dependent bias model, but rather it is a constant bias (variance) model. On the other hand, parameter uncertainty increases with increasing wash-off fraction as the variance of parameter uncertainty proportionally increases with mean prediction. The parameter uncertainty accounts for a maximum of 0.06 (6%) wash-off fraction when 95% predictive interval is considered.

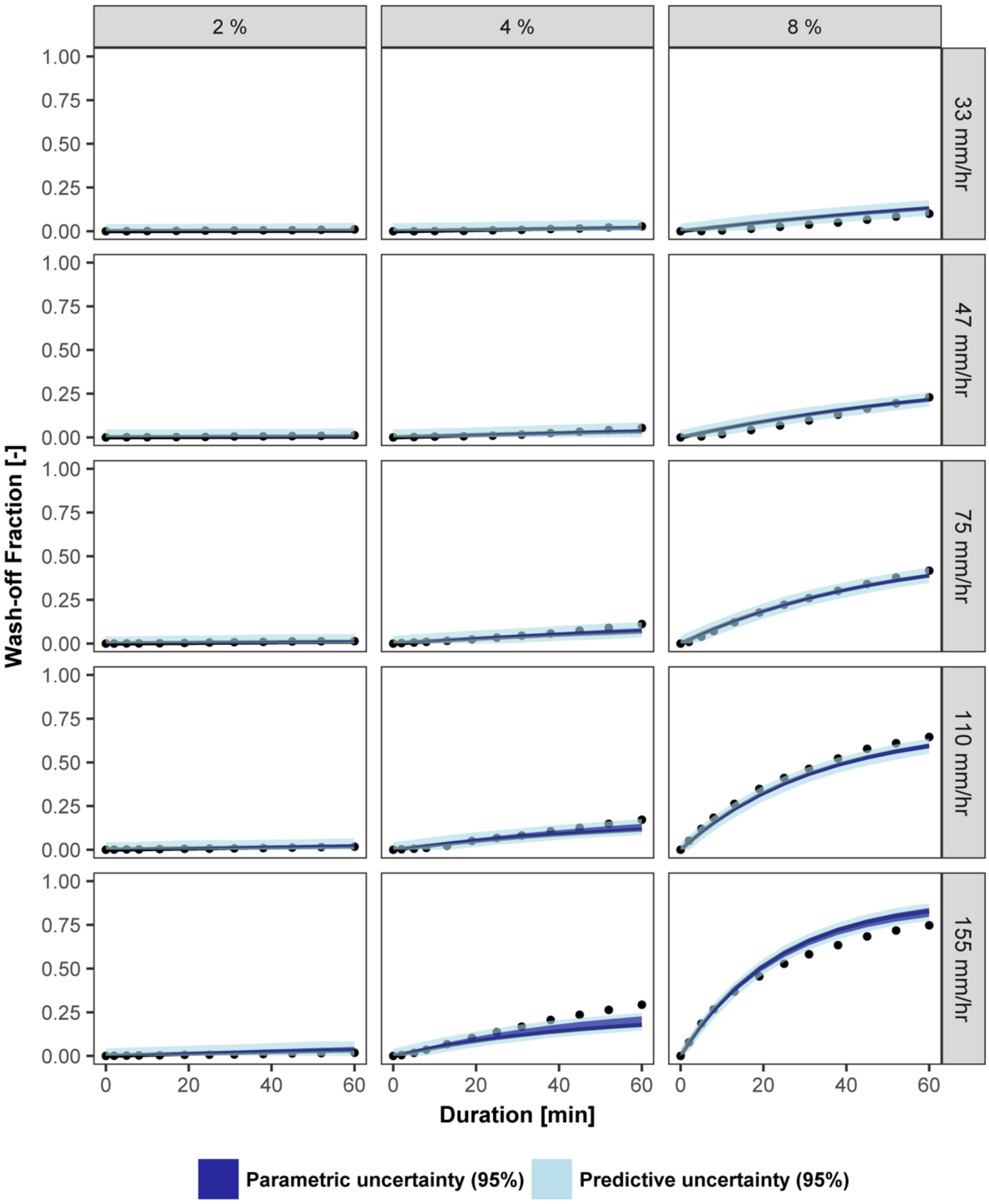

Figure 5: Uncertainty associated with the estimation of wash-off fraction using NEI

To check the reliability of the uncertainty estimation, prediction interval coverage probability (PICP, Ref Eq.8) which measures the probability that the observed values lie within the estimated prediction intervals (Shrestha and Solomatine, 2006) was used.

$$PICP = \frac{1}{n}\sum_{i=1}^{n} R * 100\% \qquad where\ R\begin{cases}1, & PL_t^u \le O_t \le PL_t^l \\ 0, & otherwise\end{cases} \tag{8}$$

Where, PL_t^u,PL_t^l are upper and lower boundary of the considered prediction interval at time *t* for a given slope and rainfall intensity, O_t is corresponding measured wash-off fraction at time *t*. For a better performance, PICP should be close to the considered prediction interval, which is 95% in this case. The calculated PICP during validation stage is 82%, so the corresponding accuracy of the uncertainty estimation is around ~ 85% which essentially means that the error model is able to predict the uncertainty reasonably well.

3.4 General discussion

IID is the most commonly used form of error model in urban hydrology (Breinholt et al., 2012; Dotto et al., 2011; Freni et al., 2009; Sage et al., 2015) mainly because of its simplicity. However, it requires the absence of a serial correlation in the error distribution, which can lead to underestimation of uncertainty and biased parameter estimates (Del Giudice et al., 2013). Error process of hydrological phenomena, such as sediment wash-off, are shown to be temporally auto-correlated and assumption of independence is not satisfied (Schoups and Vrugt, 2010; Sage et al., 2016). likelihood function based on uncorrelated error model generally leads to narrower posterior probability densities, which results in overconfident parameter estimates and unreliable uncertainty intervals. An autoregressive model helps in preventing such biases both during inference and prediction. Unlike IID, where each data point is a sample of the error distribution, an autoregressive process takes the whole time series of errors as one sample realization of the process, (in an n-dimensional space), therefore avoiding overconfidence in parameter estimation. For example, Sage et al. (2015) acknowledged that their assumption to model error process associated with wash-off modelling as IID was found to be invalid. Further, Sage et al., (2016) showed that the use of IID to represent the structural deficit of sediment wash-off models violates the statistical properties of the structural deficit and it may result in unreliable estimation of model parameters and total predictive uncertainty. An autoregressive model accounts for this autocorrelation of the error and hence, it represents the structural deficit better.

This error model can be further improved by accounting for non-normality of the structural bias. However, this adds more complexity and such added complexity could be an acceptable compromise when there is a very large number of data points to learn about the error model parameters. In our case, the current description of errors seems adequate as suggested by verification measures that show around 85% accuracy of the error models in capturing the uncertainty. Further, we assumed a constant bias to keep this autoregressive error model simple. Nevertheless, it is also possible to describe it as an input – dependent bias (Del Giudice et al., 2013) where bias can be a function of both slope and intensity. The advantage of such bias description still needs to be investigated in the uncertainty analysis of wash-off modelling.

Note that in addition to rainfall intensity and surface slope, other parameters such as sediment size and surface texture will also affect the sediment wash-off, but due to the limitations in the data used in this study, the NEM does not include the effect of these parameters. With smaller sediment sizes and smoother surfaces, the wash-off is expected to be higher. For example, Egodawatta et al. (2007) in a similar experimental study used a larger range (0 – 1000 µm) sediment resulting in a relatively higher wash-off fraction. Further, Hong et al., (2016) in their studies used a sediment range of 0- 400 µm and showed that most (> 90%) of the finest particles are removed at the beginning of a rainfall event, with about 10%–20% of medium-size particles are removed over the later part of the even. These studies show that selection of sediment size affects the sediment wash-off process significantly. Hence, the application of the NEM needs to be checked against different sediment sizes and also against different surface textures. It is expected that the values of c_1,..,c_4 will be different for different particle size distribution of the road sediment and/or different surface roughness. The inclusion of the effect of these parameters explicitly might introduce more complexity in the equation, nevertheless, such an equation can be applied globally regardless of individual catchment conditions. This is one of the research areas in sediment wash-off modelling that requires to be investigated in detail.

While experimental set-ups like the one used in this study give great flexibility to replicate the real hydrological processes such as sediment wash-off, there are still some limitations which need to be taken into account. The exponential wash-off model was improved based on experimental results which were obtained from rainfall events with constant rainfall intensity throughout the duration of an event. Keeping the rainfall intensity constant makes it easier to understand the physical wash-off process and to consequently modify the wash-off model. In fact, most of the previous studies used a constant intensity rainfall event to understand the wash-off process and consequently apply the results to develop and improve the wash-off equations. These studies include Sartor and Boyd (1972) where the exponential model was originally proposed and Egodawatta et al. (2007) where the capacity factor was first introduced in the exponential wash-off. However, constant intensity rainfall events are never the case in reality. Nevertheless, equation proposed by Sartor and Boyd (1972) and consequent refined version (e.g. Egodawatta et al., 2007) were all shown to be applicable for real case studies too. For example, Brodie and Egodawatta (2011) on a follow-up study on Egodawatta et al. (2007) showed that the use of mean rainfall intensity of real a rainfall event as a representative intensity to derive C_F produced reliable predictions. In this regard, application of NEM also needs to be checked against wash-off events resulted from real rainfall events. Such validation also needs information about surface slope.

It can also be noted that the rainfall intensities used in this experiments are generally high compared to rainfall intensities observed in the real world. However, the minimum intensity of ~ 30 mm/hr was chosen based on the trial experiments to produce measurable sediment wash-off amounts from the surface. For example, at 2% slope, even the rainfall intensity of 155 mm/hr produced only 6g wash-off total wash-off at the end of 60 min. In addition to selected sediment size and surface roughness, surface size also a deciding factor in the amount of washed off sediment as the larger surface will have a proportionally higher initial sediment load. On the other hand, unlike sediment size and surface roughness, surface size does not affect the underlying physical process and as a result, the wash-off fraction (= washed off load/initial load) will remain same. This provides the flexibility in choosing the surface size for similar wash-off experiments. The small surface size such as the one used in this study (1 × 1 m2) provides a degree of flexibility to change the experiment conditions (e.g. surface slope, initial load) and makes it possible to run such a large number of experiments. Also, it helps to keep the rainfall intensity fairly uniform over the surface. Similar sized experimental surfaces have been used in recent studies to take advantage of the above-mentioned points (Egodawatta et al., 2007; Al Ali et al., 2017). However, the trade-off is the physically lesser amount of washed off sediment from the surface and consequently the limitation in testing very mild rainfall conditions in these experiments. Hence, an optimal surface size needs to be chosen in future studies which take into account the flexibilities in the experimental setup and the minimum rainfall intensity that can produce a physically measurable sediment wash-off with limited measurement error. However, rainfall intensities used in these experiments are comparable to rainfall intensities used in similar previous wash-off studies. For example, Egodawatta et al., (2007) used a rainfall intensity range of 40 mm/hr - 133 mm/hr and 20 mm/hr - 133 mm/hr in their experiments to study the wash-off behaviour. Recently Al Ali et al., (2017) used a constant rainfall intensity of 120 mm/hr in similar experimental settings to study the wash-off behaviour from different surfaces. Due to the practical difficulty in covering a large range of rainfall intensity in an experimental set-up, extrapolation of the equation/model outside the experimental conditions is often used. Even the most widely used exponential model was originally developed for much narrower intensity range of 8 mm/hr – 20 mm/hr (Sartor and Boyd, 1972) and has been used widely for rainfall intensities that are well outside this range. One of the reasons why this is an accepted practice could be that the pattern of observations from previous studies indicate that the underlying physical transport process of wash-off are quite similar, even outside the experimental conditions that are tested. For instance, the inclusion of capacity factor as a function of rainfall intensity and slope would be valid for smaller rainfall intensities as even higher intensities have a maximum capacity in wash-off load as seen from the experimental results. Hence, although NEM has not been calibrated against smaller rainfall intensities, we believe the model structure of NEM would still be applicable to smaller rainfall intensities. Nevertheless, this should be verified in future studies.

4. Conclusions

In this study, we proposed an improved exponential wash-off model where a more physically realistic structure was added to the original exponential model by replacing the calibration parameters with functions of external drivers associated with catchment surface and rainfall characteristics. This improvement avoids the need for empirical look-up table/charts and interpolation/extrapolation and introduces some transparency in the parameter estimation which is otherwise a "black box" approach. Further, replacing the invariant calibration parameters with functions of external drivers (i.e. rainfall intensity and surface slope) makes it easier to investigate the propagation of errors in the external drivers (e.g. rainfall intensity) as these external drivers are now explicitly defined in the new equation. This new exponential model (NEM) was calibrated and verified using the experimental data collected for different combinations of surface slopes and rainfall intensities. Bayesian inference, which allows the incorporation of prior knowledge, was implemented to estimate the distribution of the parameters of the newly introduced functions. In addition, by statistically describing model bias and measurement noise, different sources of uncertainty in the prediction of NEM were separately estimated.

During calibration, NEM with a fixed set of parameter values performs as well as OEM which is calibrated for each and every experimental condition separately. At validation, NEM's performance improves over OEM, reflecting the ability of NEM to perform better under new catchment conditions. Verification measures show the uncertainty estimates associated with NEM predictions are plausible, indicating that the use of two error terms, autoregressive error and independently identically distributed error, to represent model bias and measurement noise respectively was a reasonable representation of the error process associated with sediment wash-off modelling. The total predictive uncertainty which accounts for both model bias and measurement noise accounts for ~ 0.1 (10%) uncertainty in wash-off fraction when 95% predictive interval is considered out of which a maximum of 0.06 (6%) comes from the parameter uncertainty.

It should be noted that the optimal values of $c_1,..,c_4$ in NEM needs to be checked against different sediment sizes and different surface roughness as these are two other major external drivers which would affect the sediment wash-off. Nevertheless, the model structure of NEM would be applicable for any sediment size and surface texture as the underlying physical processes will be the same as those on which the model structure of NEM was developed.

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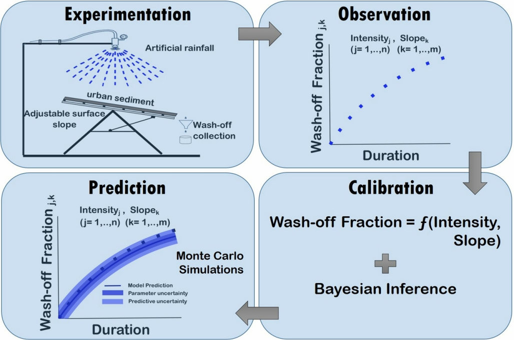
Highlights

1. An improved wash-off model, with intensity and slope as inputs, is suggested.
2. This model performs better than the original model under new catchment conditions.
3. A realistic error model is employed to quantify and decompose uncertainty.
4. Measures show that the error model used represents the uncertainty well.



Experimentation
Artificial rainfall
urban sediment
Adjustable surface slope
Wash-off collection
Observation
Intensity$_j$, Slope$_k$
(j= 1,..,n) (k= 1,..,m)
Wash-off Fraction $_{j,k}$
Duration
Calibration
Wash-off Fraction = f(Intensity, Slope)
Bayesian Inference
Prediction
Intensity$_j$, Slope$_k$
(j= 1,..,n) (k= 1,..,m)
Wash-off Fraction $_{j,k}$
Monte Carlo Simulations
Model Prediction
Parameter uncertainty
Predictive uncertainty
Duration